The Sorites Paradox and Vagueness

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The Sorites Paradox

Sorites (*σωρείτης*) is a derivation of *soros* (σωρός), the Greek word for "heap". The paradox has a long history, originating possibly from Eubulides of Miletus in the 4th century B.C.. [1] The nature of the paradox is a syllogism where a series of propositions develop through small increments such that the predicate of each becomes the subject of the next eventually yielding a paradoxical conclusion, formed of the first subject and last predicate. The paradox arises because the predicates involved in sorites syllogisms have an indeterminacy of application, highlighting vagueness in language usage and the perils of following some seemingly innocuous logical steps.

So what is the sorites paradox? In its original form, and the one that gave its name, the sorites paradox focused on the predicate "a heap" and when it has application. Start with what we accept as a heap consisting of 10,000 grains of wheat or sand and remove one grain. What remains, a collection of 9,999 grains, is still a heap because the removal of just one grain does not turn a heap into a non-heap. This illustrates what is known as the *tolerance* of vague predicates, that the application of the word is unaffected by small changes. But repeat that 9,999 times and you have nothing left. At each step the removal of one grain appears to make no significant difference and yet finally something significant has occurred. The question is when did the removal of one grain change the heap into a non-heap?

The problem of the paradox does not arise through the concept of "grain". Of course, grains of wheat, sand and others differ from each type and between examples of the same type with respect to size, shape, weight, colour, texture, etc.. Even though there is vagueness with regards to real objects this is not the source of the problem driving the paradox because in the sorites paradox a "grain" is treated as "a unit of measure" and in this sense it is not vague, but precisely described.

The paradox arises through the meaning of a "heap". What is a heap? The Shorter Oxford English Dictionary gives one definition as: *A collection of things lying one upon another so as to form an elevated mass roughly conical in form.* [2] This definition illustrates the vagueness of language. How *many* things? How *high* is an elevated mass? How *roughly* is the form conical? There is an indeterminacy, no sharp boundary, in the concept of a heap that does not allow one to mark off precisely the point when the removal of one extra small grain makes something a heap rather than a non-heap. The vagueness is not due to our failing to have information about the collection of grains for we can count the number of grains and come to know the precise number; we can scan and measure the shape of the collection to make a three-dimensional representation; and we could weigh the collection of grains. We could know all this information and still find ourselves in a situation where we would be unsure whether the predicate "is a heap" has application.

Solutions

How can we solve the sorites paradox? For any syllogism there are at least three options:

Accept the argument and the conclusion.
Reject the reasoning of the argument.
Reject one or more of the premises.

Accept the Argument and Conclusion

One option is to accept the argument and the conclusion and say that there is no paradox. The problem is dissolved. Who would take such a point of view? Now, this does seem strange but let us take a look at the argument more closely and analyze what the implications are.

(1) A collection of 10,000 grains is a heap.
(2) If a collection of 10,000 grains is a heap, then so is a collection of 9,999.
(3) If a collection of 9,999 grains is a heap, then so is a collection of 9,998.
…
(10,000) If a collection of 2 grains is a heap, then so is a collection of 1 grain.
(10,001) If a collection of 1 grain is a heap, then so is a collection of 0 grains.
(10,002) If a collection of 0 grains is a heap, then so is a collection of –1.

The first thing to note is that by accepting the sorites syllogism you have to accept that any number of grains constitutes a heap. There is no number of grains which is not a heap, even when we arrive at 1 grain or 0 grains or even –1 grain. This result is bizarre and goes against our ordinary standards of language usage. By accepting the sorites syllogism one would have to radically revise our understanding of language and the concepts that we currently live by and that would be too much to give up for one paradoxical syllogism.

A second problem is that it leads to a contradiction. Sorites syllogisms can have both negative versions, like the one above where the conditional subtracts at each stage, or positive ones, where the conditional adds. Constructing a positive version of the sorites paradox for a heap gives us the following:

(1) 1 grain is not a heap.
(2) If 1 grain is not a heap, then 2 grains are not a heap either.
(3) If 2 grains are not a heap, then 3 grains are not either.
…
(10,000) If 9,999 grains are not a heap, then 10,000 grains are not a heap either.

Thus on the positive version the result is that there is no number of grains that make a heap, which contradicts the negative version, which found that there was no number of grains that was not a heap. Thus accepting the argument and conclusion of sorites syllogisms one has to accept the contradiction of there being both any number of grains and no number of grains that make a heap and this is incoherent. For these reasons the sorites syllogism cannot be accepted and a solution to the paradox has to be found.

Reject the Reasoning of the Argument.

Does the paradox arise due to a fault in the logic of the argument? Starting with an initial, categorical premise, *modus ponens* reasoning (given p, if p then q, so q) is applied and employing *cut* each subsequent sub-argument is chained together so that the argument progresses through a series of conditional premises. These logical rules are valid according to the standards of classical logic and so the reasoning cannot be rejected and is found to be sound.

However, it might be contended that because vague lexical items have tolerance although it is legitimate to apply *modus ponens* reasoning over a short number of repetitions it is not legitimate to repeat it a large number of times. But this then denies one of the fundamental principles of logic –

the transitivity of validity.[3]

Is *modus ponens* even permissible for vague concepts? Bertrand Russell said, "The law of the excluded middle is true when precise symbols are employed, but it is not true when symbols are vague…"[4] Thus it is inappropriate to use the reasoning employed in sorites syllogisms for vague concepts such as "heap". Accordingly the sorites argument is not a valid argument because by containing vague terms the conclusion can neither be valid or invalid. As Russell was trying to show that all language is vague it would mean that *modus ponens* could not be used at all. Given our real world situation and the use *modus ponens* gives us in making inferences and constructing proofs it should be admitted as a valid form of reasoning for sorites arguments.

Reject One or More of the Premises

Can any solution be found regarding the premises? The first, categorical premise of the negative construction could be challenged by saying that 10,000 grains are not sufficient to make a heap. That debatably might be true, but misses the point that 10,000 is an arbitrary number which can be raised by any factor to make a collection of grains that everyone agrees, in common usage, to be called a heap. Conversely, regarding the first premise of the positive construction, 1 grain surely cannot be a heap as it is singular, and as we have seen by the dictionary definition given earlier a heap is a collection, a plurality, of things. Thus the first premise for both the negative and positive constructions can be accepted.

What about the subsequent, conditional premises? Can they, too, be accepted or should they be rejected? It seems uncontroversial to agree with the second premise of the negative construction that *if a collection of 10,000 grains is a heap, then so is a collection of 9,999* because having agreed that 10,000 grains are a heap the removal of one is insignificant in turning the collection from being a heap into one being a non-heap, demonstrating the idea of *tolerance*, that concepts such as "is a heap" have extensional applications. However, the 10,000th premise, *if a collection of 2 grains is a heap, then so is a collection of 1 grain*, as well as the 10,001st and 10,002nd premises surely need to be rejected. 1 grain fails to be a heap because it is singular, as does -1 grain. Can you have negatively valued heaps? In mathematics and physics maybe, but what about everyday life? In any case -1 is singular and thus cannot be called a heap and surely 0 grains cannot be a heap as zero is nothing. Thus one, or more, of the intervening premises has to be rejected, but which one, or ones?

The problem lies with the meaning of "heap". Is a collection of two grains a heap? If the two grains are lying next to each other and touching, is that a heap? What if you have three or more touching but lying on a horizontal axis? Is that a heap? Surely one of the concepts of a heap is to have a three-dimensional formation. So, are two grains, one placed upon another, a heap? But are two grains too few to be called a heap? Mark Sainsbury notes that using the concept of a heap to illustrate the problem of sorites syllogisms maybe badly chosen because arguably you could make a heap with just four grains as it would be a stable structure without using adhesives.[5] How does ordinary language, and dictionary definitions, cope with such situations? As we saw with the dictionary definition the meaning of heap details no sharp boundaries when a collection of grains is or is not a heap and hence there is vagueness.

One way to solve the problem, which was hinted above, is to define exactly how many grains constitute a heap and hence provide a clear boundary for when the predicate "is a heap" has application. For example we could define a heap as being a collection of 4 grains. Thus any collection of 4 or more grains is a heap and any collection of 3 grains or fewer is not a heap. Using the principle of bivalence, either the collection of grains is a heap or it is not a heap, and having now clearly defined what we mean by a heap we can easily see which of the intervening premises in the sorites syllogism can be accepted and which need to be rejected and hence the exact point where the argument fails – we have identified the conditional premise where the removal of one grain does indeed make a difference to whether a collection of grains is a heap or not. The paradox has been solved. But this is not how we use "is a heap" in our everyday language. Do we really want to say that a collection of 4 grains is a heap? But, if someone was adamant that a collection of 4 or more grains does in fact constitute a heap then the sorites paradox for "is a heap" has been solved. But if we change the predicate to "is a *large* heap" then what number of grains will define this predicate? We could define a large heap as a collection of 2,000 grains, for example. Thus any collection with 2,000 or more grains would be a large heap and any collection of 1,999 or fewer grains would not be a large group. The problem is that the definition is too precise for the wide range of applications "is a large heap" has in our daily usage. Also, the value used to define the boundary is arbitrary and prescriptive. Why 2,000 and not 1,500 or 5,000 grains, or any other number? There was some rationale for selecting 4 to be the number that defines "a heap" but by what rationale do we select the number of grains to constitute "a large heap"? Further, at the boundary of 2,000 grains it seems artificial to say that by removing one grain the large heap suddenly becomes a small heap. Once again the sorites paradox arises with regards to the predicate "is a large heap".

Despite these issues Timothy Williamson[6] has forcefully defended the notion that there is indeed a last cut-off point where the removal of just one grain does make a difference between a collection of grains being a heap or a non-heap – it is just that we do not know where that cut-off point is: vagueness is ignorance, or more precisely *inexact knowledge*. Knowledge is of the essence and his account is thus epistemic. According to this view words do have sharp boundaries of application, but with regard to vague lexical items we have inexact knowledge of their boundaries of application, unlike non-vague lexical items in which we do.

How can the epistemic account claim there is a clear boundary and yet not know where it is? According to Williamson, 'Where our knowledge is inexact, our beliefs are reliable only if we leave a margin for error" (p.226) which is then formalized into a Margin of Error Principle: 'A' is true in all cases similar to cases in which 'It is known that A' is true (p.227). Thus, a borderline case 'A' of being a heap is accepted as being a heap, with a margin of error, if it is very similar to other cases in which we are certain are heaps. However the degree and kind of similarity depend on the situation. What degree of similarity is suitable for determining the boundary for when a heap becomes a non-heap? No information is given. Vagueness is a source of inexact knowledge and thus the Margin of Error Principle applies. We need to know if the predicate "is a heap" has application or not, for example, "1,999 grains make a heap." In order to assert this we first need to know that 2,000 grains are a heap, which is taken that we do, and because 1,999 grains are sufficiently similar to 2,000 grains, because there is a difference of only one grain, we can rightly say, with a margin of error, that 1,999 grains are too. Williamson formularizes this: (!) If we know that *n* grains make a heap, then *n -1* grains make a heap (p232). This basically is the logic that drives the sorites paradox. How does this explain the ignorance we have of the supposedly sharp boundary of application that vague predicates have? According to the epistemic account there must be a point where *n* grains is a heap and *n -1* grains is not a heap. Now given (!) above Williamson says we cannot know the conjunction, the dividing line between being a heap and a non-heap, because to know this point we would have to know its first conjunct (2,000 grains make a heap), but by (!) the second conjunct (2,000 *n -1* grains make a heap) would be false, thereby making the conjunction unknowable. We are ignorant of the cut-off point for a collection of grains being a heap despite the fact that "heap" has a sharp boundary.

Is this tenable? How can an epistemologist assert definitely that there is a boundary and yet not know where to draw the line? How do I know 2,000 grains make a heap? Because I know 2,001 grains are. How do I know 2,001 grains are? Because I know 2,002 are. Where do you draw the line and say for certain that a collection of *n* grains constitute a heap? When you know this particular *n* and then follow (!) how do you know when to stop? Rather than being a margin of error there appears to be a wide, or even total, margin of error. Surely the sorites paradox has not been solved.

Conclusion

The sorites paradox remains. An epistemic approach should not be abandoned however as it does have the advantage of retaining classical logic with its concept of bivalence. My brief critique above is too short but I do question the idea of there being any sharp cut-off point for vague predicates. There are degrees of usage and hence many-valued logics might be more appropriate in dealing with such a spectrum.

Notes

1 Hyde, D. 2005. p. 2.

2 Vol. 1, p.938.

3 Keefe, R. (2000) p. 20.

4 Russell (1923) pp. 62/3 in Vagueness: A Reader, Edited by Keefe, R. and Smith,P

5 Sainsbury, M. *Heap, paradox of the* entry in The Oxford Companion to Philosophy. See also Sainsbury, R.M. and Williamson, T. 1997. p. 480.

6 The following page numbers are taken from his book *Vagueness*.

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