

New ansatz for neutrino mass matrix with USY

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Abstract

We propose an extension of the Standard Model (SM) containing new ansatz on the structure of Yukawa couplings with Universal Strength of Yukawa couplings (USY) for lepton sector. It describes that the left-handed Majorana neutrino mass exists, the Dirac mass for neutrino is the same as that for charged lepton sector, and the strength of vector-like charged lepton couplings with singlet scalar is the same as that of vector-like down-type quark. Under this ansatz the mass hierarchy between neutrino and charged lepton can be explained. Then the neutrino mass matrix can be reproduced.

1. Introduction

At present, phenomena of particle physics are almost understood within the SM[1]. However there are many problems in the SM model, particularly the determination of Yukawa couplings is one of the most salient ones. In order to solve this problem, many suggestions were proposed in literatures. The USY hypothesis was proposed as an interesting solution in 1990[2]. Studies of the USY hypothesis were started from the formulation of quark sector[2-9]. Then that of lepton sector was done[10-14]. The extended USY SM with extra fermions has been also investigated[5,10-12].

In this article we propose a new ansatz for lepton sector in USY model with some vector-like fermions, charged lepton(E, E^c), down-type

quark(D, D^c) and right-handed neutrinos.

This article is organized as follows. In Sec 2, the USY models for the lepton sector is reviewed. In Sec 3, the model with new ansatz is presented. Sec.4 is devoted to the summary and discussion.

2. Review of USY in lepton sector.

In literatures, the models with the USY hypothesis for the lepton sector have been studied[5,10-12]. These studies are summarized in table.1. It is shown in Refs. [10-12] that the respective phases of Yukawa couplings are connected to the lepton mass ratios.

TABLE 1. The studies on the USY for the lepton sector

Reference	$c_u \ll c_t$	Dirac mass	Majorana mass	Extra lepton	Feature
This paper	Explained	○	○	Three vector-like charged leptons and three vector-like neutrinos	Possibility of $c_\nu \sim c_t - c_u - c_d$
[10]	Not	○	○	×	Special ansatz for phase structure $\Rightarrow \sin^2 2\theta_{atm} = 0.933,$ $\sin^2 2\theta_{sol} = 0.019(SMA),$ and small lepton mixing
[5]	Not	○	×	Three vector-like charged leptons and three vector-like neutrinos	$SU(4)_{eL} \times SU(4)_{eR} \times SU(2)_L \times SU(2)_R \times U(1)_X$ symmetry breaking.
[11]	Not	×	○	×	Seesaw mechanism in neutrino sector $\Rightarrow \sin^2 2\theta_{atm} = 0.884,$ $\sin^2 2\theta_{sol} = 1.0(LMA),$ and large lepton mixing
[12]	Not	○	×	Fourth ordinary neutrino	Small phase in Yukawa couplings and large lepton mixing

We review the diagonalization of a mass matrix with USY. A 3×3 democratic ordinary fermion mass matrix is given by

$$M = c \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (2.1)$$

It is diagonalized as

$$M \rightarrow F^T M F = M_{diag} = c \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad (2.2)$$

where

$$F = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1 & 2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}. \quad (2.3)$$

Then two zero eigenvalues and one nonzero eigenvalue are obtained. Because of the two degenerate eigenvalues, the first two columns in F can be transformed by any 2×2 orthogonal matrix. According to the experimental data, all the fermions have nonzero masses except for the lightest neutrino, which has not been constrained yet by any experiments. Hence the flavor democracy should be broken. In the USY scenario, this democracy is broken by the phases of the elements of Yukawa matrix without breaking the democracy of the magnitude of Yukawa couplings. Then, a mass matrix in the USY scheme is given as

$$M = c \begin{bmatrix} e^{i\phi_1} & e^{i\phi_2} & e^{i\phi_3} \\ e^{i\phi_4} & e^{i\phi_5} & e^{i\phi_6} \\ e^{i\phi_7} & e^{i\phi_8} & e^{i\phi_9} \end{bmatrix}. \quad (2.4)$$

The masses of the first two generations of fermions are generated due to the small phases ϕ_n . By the use of USY mechanism, it is explained that the third generation of fermion except for the neutrinos is much heavier than the first two ones.

On the other hand, in the neutrino sector it is shown by the data of the Super Kamiokande experiment that the three neutrinos have small nonzero mass-squared differences. An example of 3×3 mass matrix which provides three degenerate eigenvalues is given in Refs.[10-12] as

$$M_\nu = c_\nu \begin{bmatrix} e^{\frac{2}{3}\pi i} & 1 & 1 \\ 1 & e^{\frac{2}{3}\pi i} & 1 \\ 1 & 1 & e^{-\frac{4}{3}\pi i} \end{bmatrix} . \quad (2.5)$$

The M_ν is diagonalized as

$$U_\nu^T M_\nu U_\nu = c_\nu \times \text{diag}(e^{\frac{2}{3}\pi i} + 1, e^{\frac{2}{3}\pi i} + 1, e^{\frac{2}{3}\pi i} + 1) , \quad (2.6)$$

where

$$U_\nu = \frac{e^{-\frac{\pi}{4}i}}{\sqrt{3}} \begin{bmatrix} e^{\frac{2}{3}\pi i} & 1 & 1 \\ 1 & e^{\frac{2}{3}\pi i} & 1 \\ 1 & 1 & e^{\frac{2}{3}\pi i} \end{bmatrix} \cdot K , \quad K = \text{diag}(-1, 1, 1) . \quad (2.7)$$

The specific features in this example are $c_\nu \ll c_l$ and the large phases which break the flavor democracy of Yukawa couplings.

Moreover, two specific cases of ansatz for the phase structure of the lepton mass matrices are proposed in Ref. [10,12].

One[10] is as follows,

$$M_l = c_l \begin{bmatrix} e^{-ia} & 1 & 1 \\ 1 & e^{ia} & 1 \\ 1 & 1 & e^{ib} \end{bmatrix}, \quad M_\nu = c_\nu \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{i\beta} \end{bmatrix} \quad (2.8)$$

where $a = 3\sqrt{3} \frac{\sqrt{m_e m_\mu}}{m_\tau}$, $b = \frac{9 m_\mu}{2 m_\tau}$, $\alpha = \frac{2}{3}\pi$, and $\beta = \frac{2}{3}\pi$. This ansatz provides a large angle solution for the atmospheric neutrino problem and a small angle solution for the solar neutrino problem. However the small angle solution is not favored by the experimental results of the Super Kamiokande, Kamland and SNO.

The other[12] is as follows,

$$M_l = c_l \begin{bmatrix} e^{ia} & 1 & 1 \\ 1 & e^{ia} & 1 \\ 1 & 1 & e^{-i(a+b)} \end{bmatrix}, \quad M_\nu = c_\nu \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{-i2\alpha} \end{bmatrix} \quad (2.9)$$

where $a = 3 \frac{m_e}{m_\tau}$, $b = \frac{9 m_\mu}{2 m_\tau}$, and $\alpha = \frac{2}{3}\pi$. This ansatz can explain the large

angle solutions for both the atmospheric and solar neutrino problems.

To explain the hierarchy for $c_\nu \ll c_l$, a 6×6 neutrino mass matrix $M_{\nu 6 \times 6}$ with USY is studied[12]. Then, the neutrino mass matrix is given as

$$[v_L^T \ v_R^T] \begin{bmatrix} 0 & m_D \\ m_D^T & M_R \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix} \quad (2.10)$$

where

$$m_D = \lambda \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad M_R = \mu \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (2.11)$$

$M_{\nu 6 \times 6}$ can be diagonalized as,

$$\begin{bmatrix} m_{eff} & 0 \\ 0 & M_R' \end{bmatrix} \quad (2.12)$$

where

$$m_{eff} \approx -\frac{\lambda^2}{\mu} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad M_R' \approx \mu \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (2.13)$$

Hence, the existence of large right handed neutrino Majorana mass with $\mu \gg \lambda$ generates one small but nonzero mass for the ordinary neutrinos.

On the other hand, an alternative mechanism without resorting the seesaw mechanism which generates three extremely small neutrino masses is proposed in Ref. [11]. It requires the existence of fourth generation of neutrino. Then, three quasi degenerate masses near zero and one large mass are given. In this model the structure of the phase is proposed as

$$M_\nu = c_\nu \begin{bmatrix} e^{i\alpha} & 1 & 1 & 1 \\ 1 & e^{i\alpha(1+\varepsilon)} & 1 & 1 \\ 1 & 1 & e^{i\alpha(1+\delta)} & 1 \\ 1 & 1 & 1 & e^{i\alpha} \end{bmatrix}. \quad (2.14)$$

If the three phases are very small, then the very small masses of the three neutrinos can be explained. It is noted that this mechanism can be realized whether the neutrino mass is given by Dirac or Majorana mass term.

3.The model

3-1 Ansatz

A new vector-like fermion model with USY is presented in this section. E6 inspired vector-like charged leptons (E, E^c), right-handed neutrino and vector-like down-type quarks (D, D^c) are included in the model. The

quantum numbers of these particles under $SU_3(3) \times SU_L(2) \times U(1)$ are shown as

$$E \sim (1,1;-1), E^c \sim (1,1;1), \nu_R \sim (1,1;0), D \sim (3,1;-\frac{1}{3}), D^c \sim (3,1;\frac{1}{3}). \quad (3.1)$$

The reason why the top quark mass is much larger than the bottom quark one can be explained by the existence of heavy D and D^c [9]. On the other hand, in the lepton sector the problem of mass hierarchy between the neutrinos and charged leptons is more difficult to explain. Quasi degenerate neutrino masses are generated by Majorana mass term[12]. All the democratic fermion mass matrices are shown as

$$M_u \simeq c_u \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, M_d \simeq c_d \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ k & k & k & k \end{bmatrix},$$

$$M_\nu \simeq c_\nu \left[\begin{array}{ccc|ccc} x & x & x & 1 & 1 & 1 \\ x & x & x & 1 & 1 & 1 \\ x & x & x & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & y & y & y \\ 1 & 1 & 1 & y & y & y \\ 1 & 1 & 1 & y & y & y \end{array} \right], M_l \simeq c_l \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ k' & k' & k' & k' \end{bmatrix}, \quad (3.2)$$

where $m_u = m_c = m_d = m_s = m_{\nu_e} = m_{\nu_\mu} = m_e = m_\mu = 0$. Then, we assume the following conditions as ansatz.

1. The left-handed Majorana neutrino mass terms are present. (For example, these Majorana mass terms are generated by the Higgs triplet [14]).
2. The strength of c_ν is the same as that of c_l .
3. The strength of k is the same as that of k' .

3-2 The neutrino mass matrix

Here, we focus on the neutrino mass matrix. We consider the model with both the left-handed and right-handed Majorana and Dirac mass terms as the most general neutrino mass matrix. It is given by

$$[v_L^T \ v_R^T] c_\nu \begin{bmatrix} xY & Y \\ Y & yY \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix}. \quad (3.3)$$

The model with $x=0$, that is, the ordinary seesaw mass matrix was studied already[12]. In the present study we consider the model with $x \neq 0$ in general. The M_ν is made block-diagonal as

$$M'_\nu = c_\nu \begin{bmatrix} x'Y & 0 \\ 0 & y'Y \end{bmatrix}, \quad (3.4)$$

where

$$y' = \frac{(x+y) + \sqrt{(x-y)^2 + 4}}{2}, \quad x' = \frac{(x+y) - \sqrt{(x-y)^2 + 4}}{2}. \quad (3.5)$$

Then the submatrices of M_ν are diagonalized to obtain

$$M_{\nu \text{ diag}} \approx c_\nu \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3x' & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3y' \end{array} \right]. \quad (3.6)$$

Here four neutrinos remain massless. According to the experimental data of the Z decay at LEP, there are three light neutrinos which couples to the Z boson. If $x \ll 1$ and $y \gg 1$, $[M_{\nu \text{ diag}}]_{33}$ can be small and the three neutrinos coupling to the Z boson are light enough to satisfy the experimental bound. The realistic neutrino masses should be generated in the

present USY scheme. The number of available USY phases will be enough to reproduce the realistic mass matrix. This issue will be investigated in a future work.

3-3 The mass charged lepton matrix

The charged lepton mass matrix is given by

$$M_l \approx c_l \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ k' & k' & k' & k' \end{bmatrix}. \quad (3.7)$$

This is diagonalized as follows,

$$F_{lL}^\dagger M_l F_{lR} = \text{diag}(0, 0, 0, 2c_l \sqrt{k'+3}) \quad (3.8)$$

where

$$F_{lL}^\dagger = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{6} & 2/\sqrt{6} & -1/\sqrt{6} & 0 \\ k'/\sqrt{3(k'^2+3)} & k'/\sqrt{3(k'^2+3)} & k'/\sqrt{3(k'^2+3)} & 3/\sqrt{3(k'^2+3)} \\ 1/\sqrt{k'^2+3} & 1/\sqrt{k'^2+3} & 1/\sqrt{k'^2+3} & k'/\sqrt{k'^2+3} \end{bmatrix}, \quad (3.9)$$

$$F_{lR} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & -1/\sqrt{12} & 1/2 \\ 0 & 2/\sqrt{6} & -1/\sqrt{12} & 1/2 \\ -1/\sqrt{2} & -1/\sqrt{6} & -1/\sqrt{12} & 1/2 \\ 0 & 0 & 3/\sqrt{12} & 1/2 \end{bmatrix}.$$

The first three column vectors of F_{lL} and F_{lR} may be changed by any 3×3 orthogonal transformation because the first three eigenvalues are degenerate.

4. Discussion and Summary

There are a sufficient number of degrees of freedom in the USY mechanism. Eight USY phases in a 3×3 mass matrix, fifteen in a 4×4 mass matrix, and eight scale factors c 's, k , k' , x and y exist. Moreover orthogonal transformations may be made for the diagonalization matrices F 's, generating the large lepton mixings. To decrease the number of parameters in USY models, some conditions are imposed as ansatz. In the present model, we have proposed the three conditions as follows:

1. The left-handed Majorana neutrino mass terms are present.
2. The strength of c_ν is the same as that of c_l .
3. The strength of k is the same as that of k' .

We hope that they are lead by a hidden symmetry.

In the future work, we should specify the range of the phases of Yukawa couplings and that of k' , x , and y so as to reproduce the experimental lepton masses and mixings.

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